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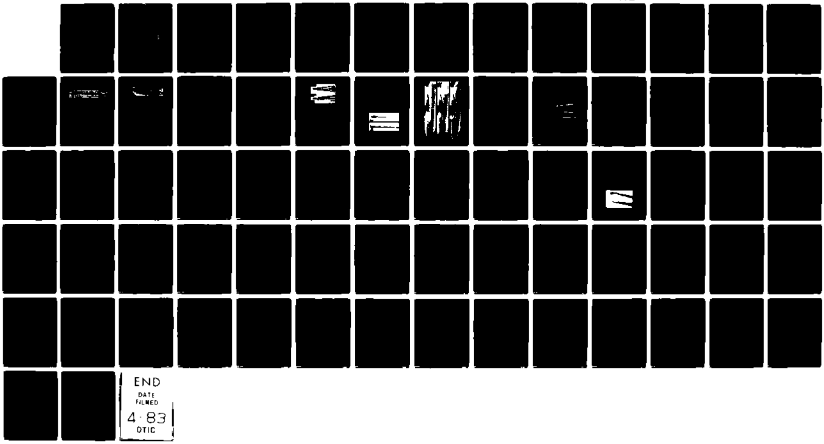

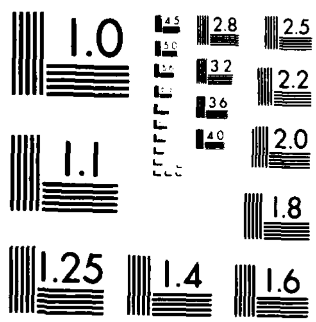
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AE&EM-TR-82-02

New Developments About Open Separation

K. C. WANG

July 1982

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New developments about open separation are discussed. A number of related experiments and calculations since 1976 are briefly reviewed and the open separation concept is seen to be overwhelmingly supported. Three-dimensional separation in general can be identified by the convergence (or running-together) of the limiting streamlines in spite that this procedure does not answer all deeper questions. An open separation is shown to start from a regular point in the middle of the surface flow field, this idea is in

20. Abstract (continued)

direct contradiction to the conventional notion of separation and has been a stumbling block for many to accept the open separation concept. The streamline vs. envelope debate remains unsettled even though there are researchers who changed their preference from the envelope to the streamline version. Criticism of open separation is replied in details. Tobak and Peake reversed their previous stand and came to a position essentially the same as ours except in terminology. Cebeci, Khattab and Stewartson objected to our open separation idea on a superficial ground even though this very idea has been supported by all related experiments and calculations. In contrast CKS' suggested alternative is a mere speculation with no evidence. Finally the open vs. closed separation concept can be carried over by analogy to unsteady cases. Our unsteady open separation idea was also once contradicted by Cebeci, but this unsteady dispute has been settled.

CONTENTS

SUMMARY

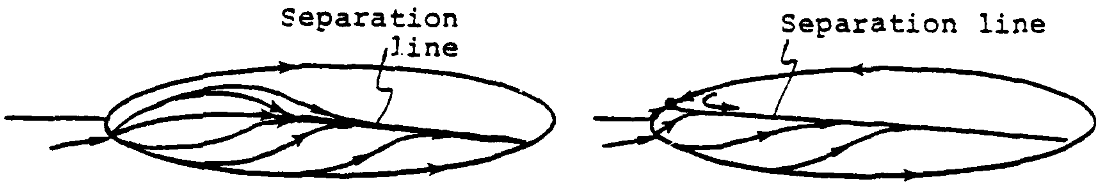
New developments about open separation are discussed. A number of related experiments and calculations since 1976 are briefly reviewed and the open separation concept is seen to be overwhelmingly supported. Three-dimensional separation in general can be identified by the convergence (or running-together) of the limiting streamlines in spite that this procedure does not answer all deeper questions. An open separation is shown to start from a regular point in the middle of the surface flow field, this idea is in direct contradiction to the conventional notion of separation and has been a stumbling block for many to accept the open separation concept. The streamline vs. envelope debate remains unsettled even though there are researchers who changed their preference from the envelope to the streamline version. Criticism of open separation is replied in details. Tobak and Peake reversed their previous stand and came to a position essentially the same as ours except in terminology. Cebeci, Khattab and Stewartson objected to our open separation idea on a superficial ground even though this very idea has been supported by all related experiments and calculations. In contrast CKS' suggested alternative is a mere speculation with no evidence. Finally the open vs. closed separation concept can be carried over by analogy to unsteady cases. Our unsteady open separation idea was also once contradicted by Cebeci, but this unsteady dispute has been settled.

1. INTRODUCTION

The open-vs.-closed separation concept for three-dimensional flows was introduced by this author[1] about a decade ago. A closed separation is consistent with the usual concept of separation carried over from two-dimensional studies, an open separation is a new idea. This new idea was evolved at first from the solutions of a symmetry-plane boundary layer[2] over an inclined body of revolution and from judicious interpretation of surface-flow experiments such as those of Werle[3] and Stetson[4]. Further supports prior to 1976 including those from both experiments and numerical solutions of three-dimensional boundary layers were reported later[5]. In the intervening years since then, a number of new developments[6-20] appeared in the literature. All except Ref. 12 lend additional supports to the open separation idea, although some[11,13] raised certain objections. In the present report, we intend (1) to review briefly those new developments in Section 2, (2) to clarify certain basic questions about open separation in Sect. 3, (3) to answer critics' objections in Sect. 4, and (4) to extend the same open vs. closed separation idea to unsteady cases in Sect. 5.

1.1 Basic Ideas of Open Separation

To begin with, we recapitulate some definitions of terminology and notations. By an "open" separation, it is meant that the separation line (Figure 1a) on the body surface is not closed in the front leeside surface and does not originate from a singular point where both skin-friction components vanish, the limiting streamlines on both sides of the separation line originate from the same front attachment (or stagnation) point.



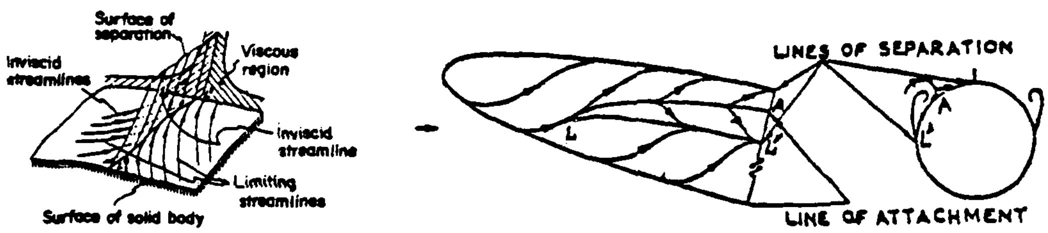
(a) Open (b) Closed

Fig. 1. Open and closed separations.

In contrast, for a closed separation, the separation line is closed in the front, it passes through the singular points of the limiting streamlines and the limiting streamlines on two sides of the separation line originate respectively from the front and the rear attachment points (or lines), Fig. 1b.

1.2 Differences with the Free Vortex-Layer Separation

The differences between Maskell's[21] free-vortex-layer separation and our open separation warrant brief comments, although this question was already addressed in Ref. 5. Referring to Maskell's sketch (Fig. 2a), it is understood that the separation line LL' contains only ordinary points,



(a) (b) (c)

Fig. 2. Maskell's sketches.

the same is emphasized for an open separation line. In this respect, there is no difference between these two. However, Fig. 2a gives no idea

to the following questions: (1) Whether the line LL' is open or closed on the front portion of separation (see Figs. 1a, b)? (2) Where the limiting streamlines on two sides of the line LL' may come from? (3) Where the line LL' begins in an actual flow? In contrast, these questions are clearly specified in the open separation concept: (1) A separation line may be open or closed with respect to the upstream limiting streamlines. (2) The limiting streamlines on both sides of an open separation line originate from the same front stagnation point or line. The situation would be different if the limiting streamlines on either side may come from a different source, for example, from the outer flow through reattachment. And (3) An open separation line begins in the mid-field of the limiting surface flow, whether this is true or not has important consequences. This is precisely one of the current disputes (Sections 3 and 4).

Maskell's conception for similar body flow was illustrated in Fig. 9 of his original report[21]. For convenience, we may refer to those sketches as Fig. 9a, b . . . f counting from the top. His Fig. 9c (reproduced here in Figs. 2b, c) could most likely be construed to resemble our illustrating model for open separation, but actually this is not so. Fig. 2b, c depicts LL' as the separation line, LA as an attachment line, these two lines meet at L. Although L is located in the middle of flow and hence is consistent with the open separation idea, yet it is conditional with the presence of the attachment line. Furthermore, the limiting flow between LA and LL' comes from the outer flow (i.e. above the surface) through reattachment rather than from the front stagnation area. Hence Maskell's Fig. 9c does not describe an open separation as we defined.

In short, in spite that both the free-vortex-layer separation and the open separation stress to contain regular points only, there are subtle differences between them. A number of specific important features which the open separation idea brought to light were not spelled out in the free-vortex-layer case.

1.3 Identification of Separation

Before we get into details about various disputes, we would like to reiterate that in all discussions of separation, it seems the most important question is still how to identify separation. There are different definitions, each is based on certain particular symptom(s) of separation, some are more mathematical, others may be more physically intuitive. Closely connected to these definitions there involves a vast range of specialized subjects: regularity, singularity, stability, accessability, calculability . . . and so on. Debates on these diverse issues sometimes tend to give one the impression that we have lost the sense of priority of the problem. The readers who simply want to find a way of determining separation may feel confused and wonder what all the debates are for?

In our opinion, the best way to identify is still the one first suggested by Eichanbrenner and Oudart[22]. The latter authors reported that three-dimensional separation are characterized by the running-together (or convergence, coalescence . . .) of the limiting streamlines. Their use of the term "envelope" was later challenged by Lighthill and has since become a subject of prolonged debate. However, the validity of such characterization itself has never been in doubt, the envelope dispute is only a separate question.

It follows directly from the above characterization of separation that the skin friction component normal to the separation line vanishes, i.e. $\tau_n = 0$ or $c_{fn} = 0$ where τ and c_f stand for the shear and the skin function. This idea has been known in literature for some time. However, unlike in two-dimensional steady boundary layer flow, vanishing of τ_n cannot be used as a separation criterion because the separation line and hence its normal direction is not known a priori in three-dimensional flows.

For many purposes the above simple characterization is all that is needed to locate separation, although, to be sure, there are deeper questions left unanswered. This characterization may be looked upon as an unsophisticated definition of separation. It is easy to understand and straightforward to apply. Predictions of separation made on this basis have never been contradicted. Later we shall see that no matter whether the separation line is finally determined to be a streamline or an envelope (Sect. 3.2), this definition is used in locating the separation. The streamline vs. envelope debate serves to settle only a deeper question.

This definition quickly identifies separation from surface flow patterns regardless whether the patterns result from experiments or calculations. It further makes no difference whether calculations are based on the boundary layer theory, thin-layer approximation or Navier-Stokes theory. The situation will be different if singularity arguments are used in determining separation, because singularity of the boundary layer theory would disappear in the other approaches. Separation is a unique physical phenomenon, its correct location should be independent of the theory used. It is true that separation determined by boundary layer

theory may not be as accurate as that determined by Navier-Stokes calculation. But to the extent that boundary layer approximations are valid, the results should be essentially in agreement. Otherwise it would be very confusing to say the least if for a given problem, one speaks of a kind of separation according to boundary layer theory and another different kind of separation according to Navier-Stokes' theory, or experimental observations.

In this report as well as in Ref. 5 our discussion of separat will be based on the surface flow pattern. "Regular" or "singular" p s are referred to those associated with the equation of the limiting str lines. The question has sometimes been raised concerning whether separation can be determined by surface flow characteristics alone? This question cannot be answered now in definite terms, however serious challenge to this approach has so far been lacking. We shall be content with this here.

Efforts also are not made to distinguish whether the separation is laminar or turbulent. Calculations cited are laminar cases, but experiments involved turbulent flows. In spite of many differences between laminar and turbulent boundary layers, general separation characteristics are nevertheless very similar in both cases. They differ in extent, but not in character. It is in fact due to this connection which justifies the authors' participation to an IUTAM symposium for three-dimensional turbulent boundary layers. Part of the material[23] discussed here was presented at that symposium[23]. Otherwise this author's research has been exclusively confined to laminar cases.

2. RECENT CONTRIBUTIONS

Most of those recent experiments and calculations after 1976 were made for an ellipsoid of revolution, the same geometry which was used in the author's original calculations. This fact makes direct comparison much easier. In some cases, the major-minor axis ratio is 1/4 which is the same as ours. In others the value of 1/6 was used.

Patel and Han[6] used colored dyes to display the surface flow pattern over an ellipsoid of revolution ($b/a = 1/4$) in water tunnel. Although the limiting streamlines were not sharp enough due to the diffusion of dye, several features were clearly demonstrated for the first time in agreement with our predictions. Unfortunately the original color pictures are too expensive to reproduce. Figs. 3a, b show their experimental

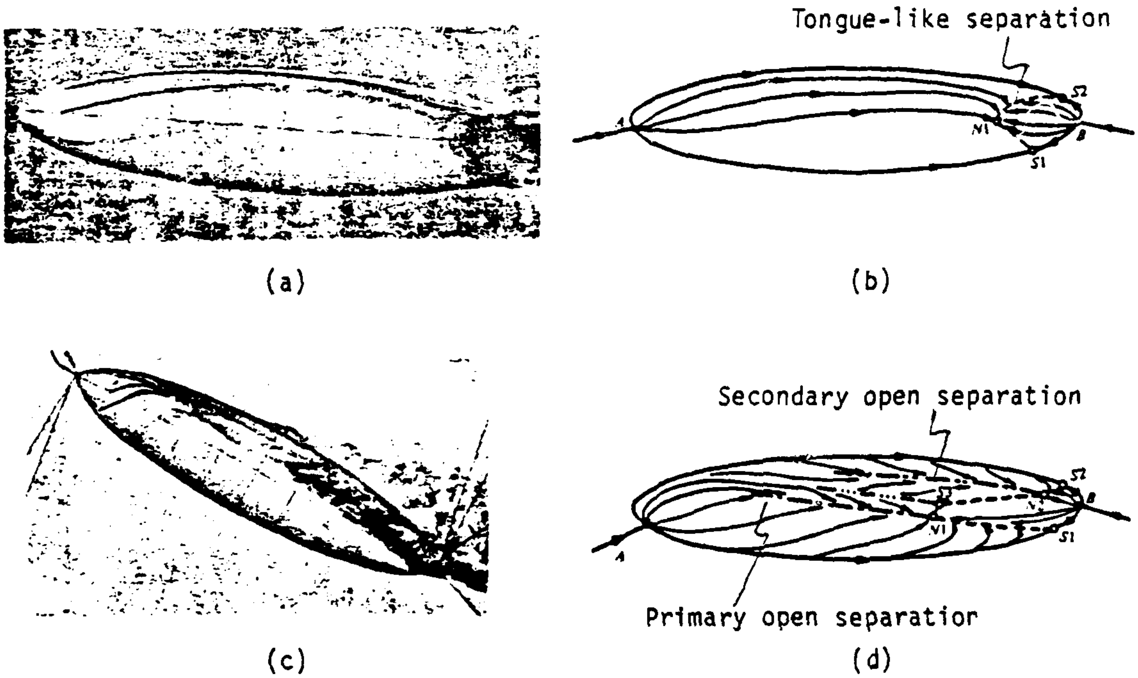


Fig. 3. Patel and Han's experiment.

picture (side view) and the corresponding sketches for a low incidence ($\alpha = 5°$) case. The latter is very close to our calculated case of $\alpha = 6°$[25]. Figs. 3c, d show the same for a high incidence case ($\alpha = 30°$), where the idea of open separation is clearly confirmed. In Figs. 3a, b, two confirmed features are worth to be pointed out: (1) the streamlines over the forebody curve downward, indicating significant area of reversed circumferential-flow ahead of separation, and (2) the separation line assumes an unusual tongue-like shape as revealed in our calculation.

More recently, Ramaprian, Patel and Choi[7] made detailed measurements of the turbulent boundary layer over an included body of revolution. Their conclusions include the following passages which reiterated their experimental confirmation on the open separation idea: "One of the primary objectives of the present experiment was to study the characteristics of the boundary layer in the neighbourhood of what Wang (1972, 1974a) has termed an open separation. . . ." "It was speculated that, in spite of the fact that the present flow is turbulent and the body geometry is different, the data would indicate the quantitative features of the flow that lead to the formation of an open separation line. This speculation, indeed, turned out to be correct as indicated by an examination of the various views of the data presented here. . . ." Their new finding is again consistent with the contention which we have held all along, i.e. the separation patterns we discussed hold for laminar as well as turbulent cases.

An extensive experimental reserach program in this subject area has been reported by DEVLR, Germany. Kreplin, Vollmers and Meir[8] studied the transitions and separation on a 1/6 ellipsoid of revolution.

Their experiments included the surface hot-film probes and the oil flow visualization. Fig. 4a shows the top view of the surface flow pattern for the case of $\alpha = 30°$. They fashioned a procedure by measuring the skin

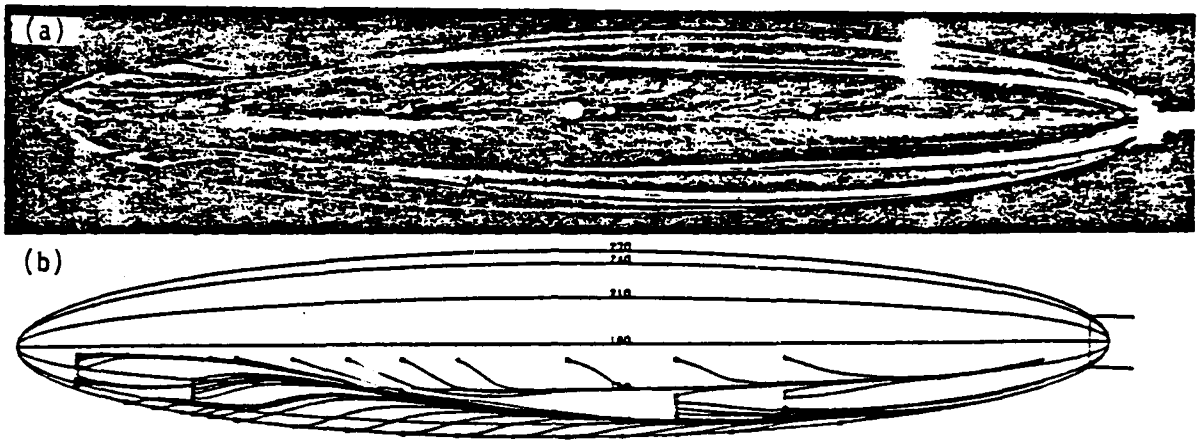


Fig. 4. Surface flow pattern due to Kreplin et al.

frictions and then calculating therefrom the limiting streamlines. The latter are shown in Fig. 4b which bears a remarkable resemblance to Fig. 4a.

Bippes[9] used the hydrogen bubbles technique to visualize the boundary layer over a 1/6 ellipsoid of revolution. By this method, the structure of the symmetry-plane flow especially becomes visible (Fig. 5). Open separation was observed to move upstream with increasing incidence, this agrees with our description in Ref. 5. Discrepancy was however reported over the question of separation jump. This discrepancy is believed to be largely due to the influence of a long support stint used in his experiment.

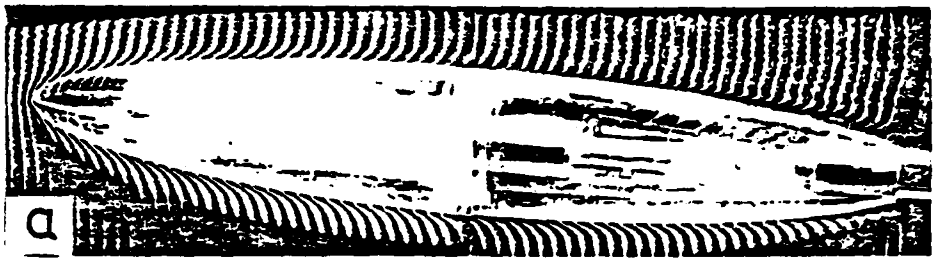


Fig. 5. Symmetry-plane flow (Bipples), $\alpha = 5°$.

During a recent visit to DFVLR-Göttingen, Dr. Bipples was kind to show this author his hitherto unpublished results. Among others, his investigation was concerned with the surface flow-patterns over a body of revolution at a large range of incidence, the results are very stimulating and more extensive than the author has ever seen before. A particular new feature which we shall mention (with Dr. Bipples' concurrence) later in Sect. 4.1 is about the nose vortices and the related separation pattern.

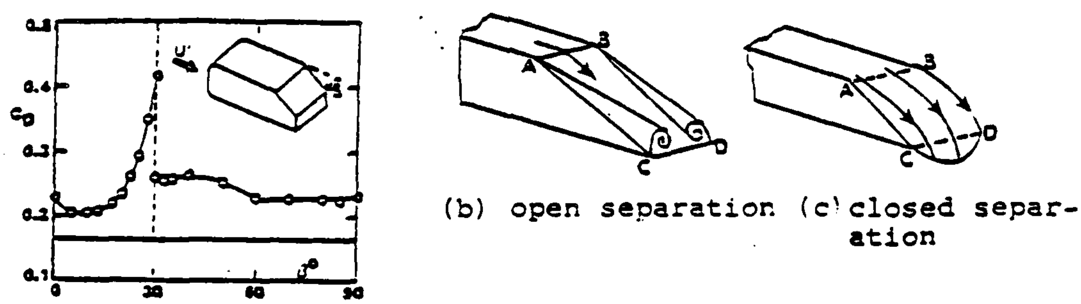
Cebeci, Khattab and Stewartson[10] reconsidered our symmetry-plane boundary layer problem[2,22] and confirmed the separation jump phenomenon we reported before. They introduced transformations which facilitated calculation near the front vertex of the body. They also demonstrated that for different values of the major/minor axis ratio, the separation jump takes place at the same critical incidence.

The same authors[11] also repeated our three-dimensional boundary layer calculations for an ellipsoid of revolution. Their results confirmed our earlier ones[24-26]. They acknowledged that open separation occurs in real flows, but they objected to speak of open separation within the context of boundary layer theory because the solution on the leeside

of the separation line could not be calculated. We shall elaborate on this question in Sect. 4.2.

Peake and Tobak attempted first[12] to contradict our open separation idea by adhering strictly to Lighhill's concept of separation. But later they[13] shifted their stand so as to be essentially same as ours, although they preferred to speak in terms of a different set of terminology. The details shall be discussed in Sect. 4.1.

An application of open-vs.-closed separation to hatch-back cars is found interesting. Morel[18] reported that the drag coefficient C_D increases as the slanting angle β increases (Fig. 6a), but at $\beta = 30°$, C_D suddenly drops and remains almost unchanged thereafter. Such a sudden drop of C_D is attributed[19] to a change-over of separation pattern from open (Fig. 6b) to closed (Fig. 6c).



(a) C_D vs β

(b) open separation (c) closed separation

Fig. 6. Separation on a hatch-back car.

Other recent contributions[14-20] will be discussed in later sections.

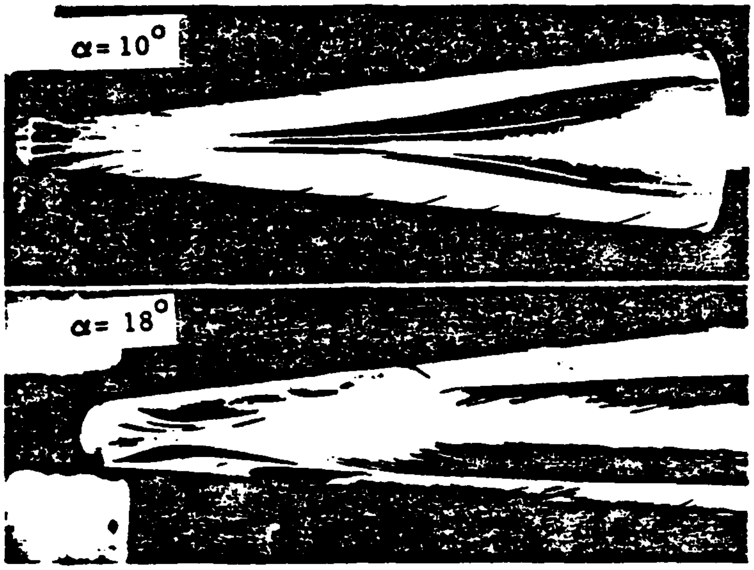
3. CLARIFICATION OF BASIC QUESTIONS

Two questions have often been raised: (1) Where does an open separation line start? and (2) Is a separation line also a limiting streamline? Or is it an envelope of the limiting streamlines?

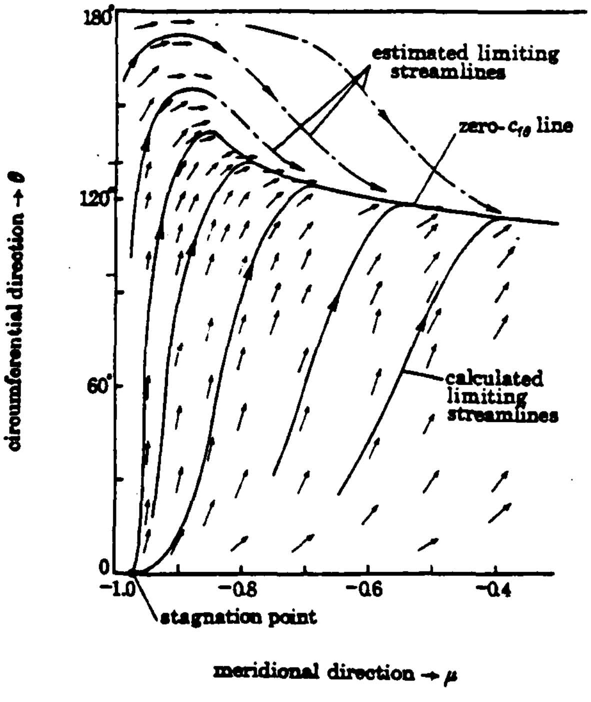
3.1 Origin of an Open Separation Line

One of the basic features for an open separation line we have indicated is that it starts from a regular point of the limiting streamline equation in the middle of the flow field. This single idea has been the main objection of its critics. This is apparently influenced by the prevailing notion of separation which maintains that a separation line cannot start in "mid-air" so to speak, instead it must start from a saddle point of singularity.

Previously documented calculations and experiments[5] did confirm some basic features of an open separation, but the evidence to ascertain where an open separation starts was not sufficient. For example, Stetson's experiments[4] for a blunt cone (Fig. 7a) shows very clearly the separation line being open in the front, but the picture does not indicate the beginning point of the separation line. Likewise the calculated limiting-flow[25] (Fig. 7b) for an ellipsoid of revolution at 30° incidence shows that the separation line is not connected to the leeside symmetry-plane (i.e. $\theta = 180°$) and hence it is open. However, the leeside calculation was very limited, so the flow structure in the vicinity of the separation starting point again was not known for sure. We shall now present evidence to demonstrate that an open separation indeed starts from a regular point in the middle of the surface flow field.



(a) Surface flow on blunt cones by Stetson(4).



(b) Calculated flow pattern by Wang(25).

Fig. 7. Previously reported open separations.

In 1968, extensive surface flow visualizations on an ellipsoid were reported by Atraghji[20], some of his results were widely quoted in the literature[5], but the specific aspect which is of particular interest to our present problem has received little attention. Figs. 9a, b, c display his surface flow patterns over an ellipsoid of revolution (b/a = 1/6) at an incidence 25°, viewed respectively from θ = 90° (Fig. 8a, i.e. side view), 135° (Fig. 8b) and 180° (Fig. 8c, i.e. top view). The primary and secondary separation lines are clearly revealed by the coalescence of the limiting streamlines. The former appears to start at point A, and the latter at point B. Central to our present concern is that both points A and B are regular points of the limiting flow. There is absolutely no sign of a singular nature.

Figs. 9a, b show the surface flow pattern over a hemisphere cylinder[28] for the Mach number being 1.2. The incidence is 10° and 15°.

(a)

(b)

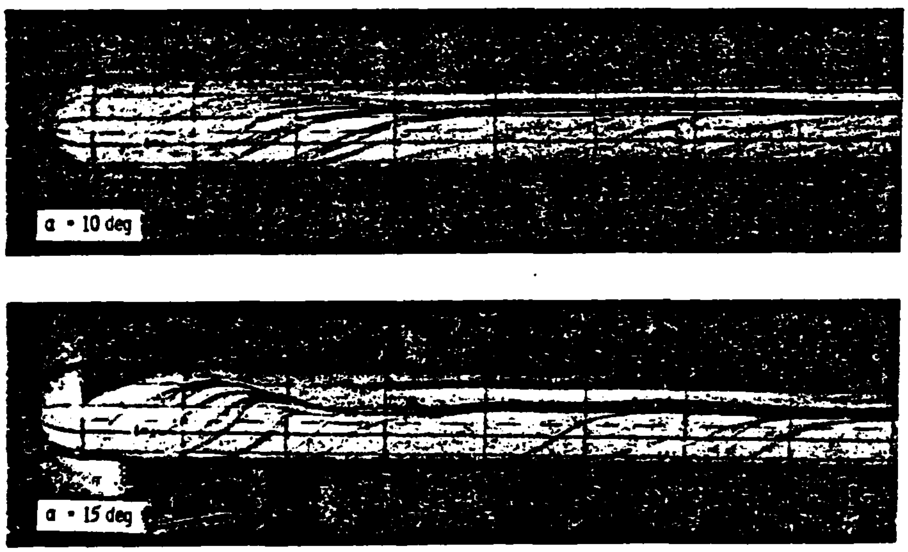


Fig. 9. Hsieh's experiments on a hemisphere cylinder.

(a)

(b)

(c)

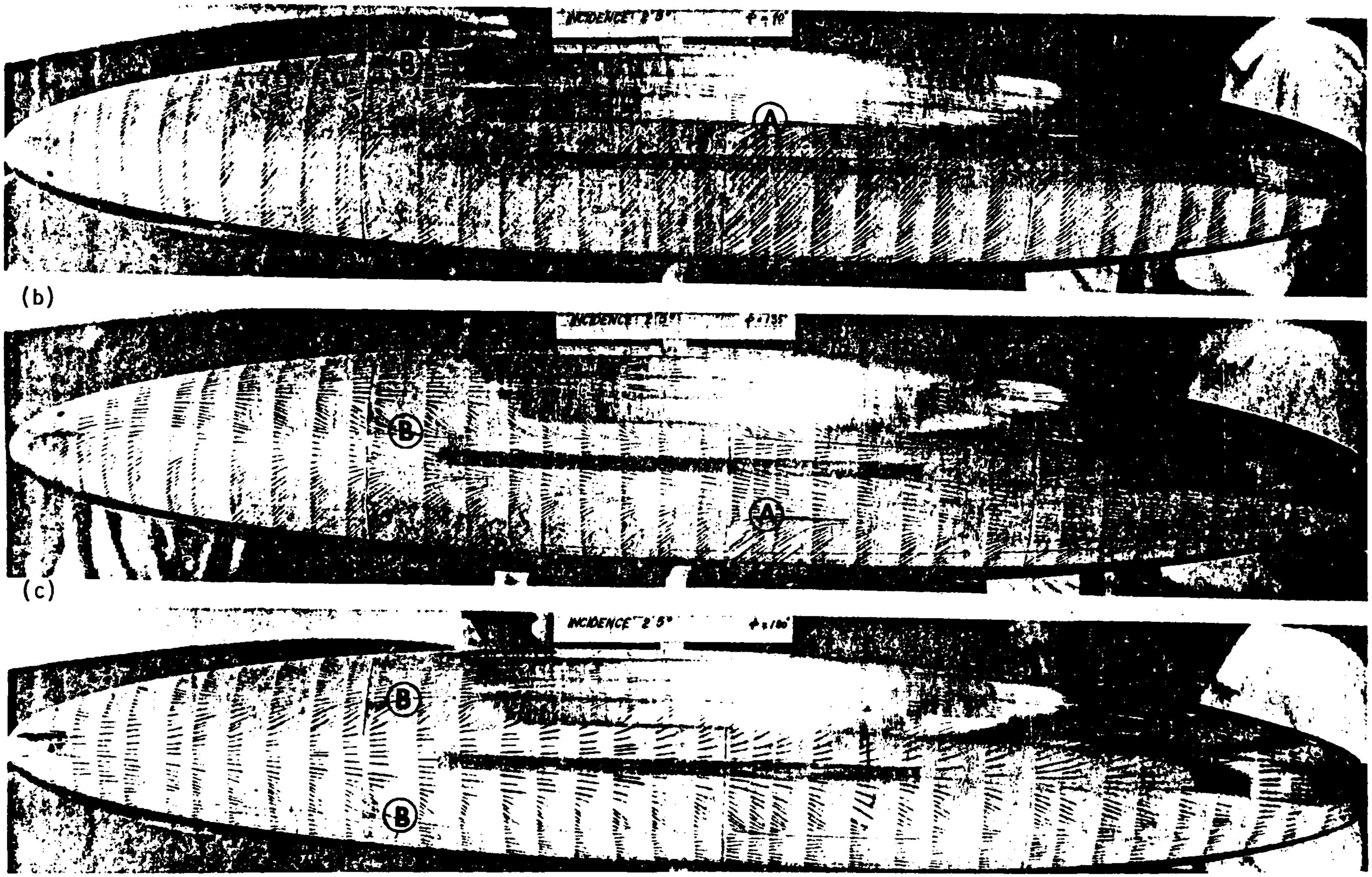


Fig. 8. Atraghji's experiments on an ellipsoid.

These pictures were quoted in Ref. 5 to represent an example of open separation and nose vortices. They are included here to demonstrate specifically the open separation line starting at a regular point of the limiting flow.

Hsieh went further[29] to calculate this type of flow using the "thin-layer" approximation. The basic ideas of the thin-layer approximation are similar to those for the boundary layer theory except that the pressure is not imposed by inviscod solutions, instead it is calculated along with the boundary layer. This permits viscous and inviscod interactions and is free from the Goldstein-singularities encountered in the boundary layer theory. The latter fact enables, among others, calculation to continue across a separation line.

Fig. 9c shows Hsieh's calculated limiting flow pattern on a hemisphere cylinder at 19° incidence. The arrows indicate the flow direction.

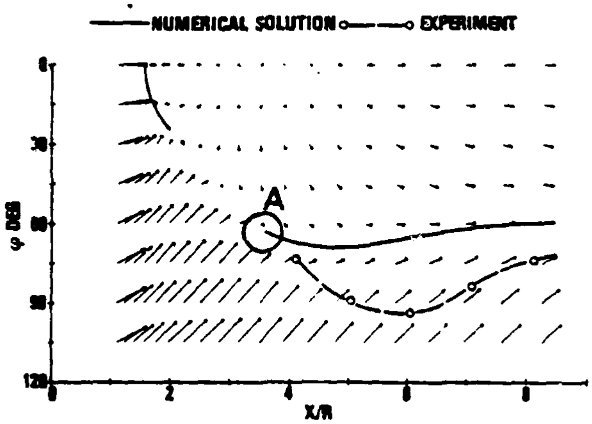


Fig. 9c. Hsieh's calculated pattern[29], $M = 1$, $\alpha = 19°$.

The calculated primary separation line differs somewhat from the experimental one, but our main interest here is that near the beginning part A, no singular pattern is noted.

Rizk, Chausse and McRae[14] reported calculated limiting flow patterns over a hypersonic sphere-cone at incidence. They used a parabolicized "thin layer" approximation. While the thin-layer approximation is time-dependent and is elliptic in space domain, the parabolicized version is a steady, parabolic-marching calculation, consequently the present work presumably provides better resolutions than Hsieh's[29].

Figs. 10a, b show the top (Fig. 10a) and the side (Fig. 10b) views of their calculated patterns. Both the primary separation line and the secondary separation lines are clearly identified by the running

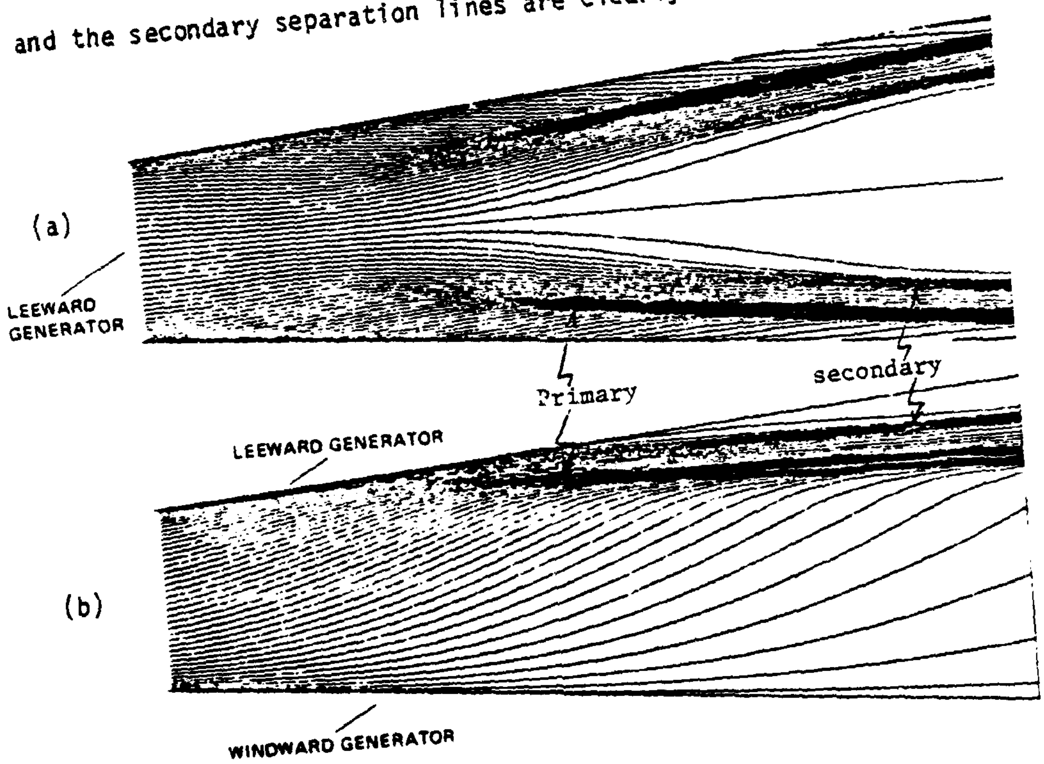


Fig. 10. Calculated pattern by Rizk et al., M = 10, $\alpha = 20°$, $\theta_c \approx 4.7°$, Re = 2.3×10^6.

together of the limiting streamlines. For our present concern, it suffices to note that the starting points of the separation lines are again

just regular points, there is no sign of any singular pattern. The authors stated that one objective of their work was to test the open separation idea. The flow patterns in Fig. 10 certainly attest to their success.

To sum up, an open separation line indeed starts at the mid of flow field (as opposed to any singularity), this is supported by both experiments and calculations in spite that this very idea is in direct contradiction to the usual notion of separation.

3.2 Envelope vs. Streamline

We next address to the second question raised at the beginning of this section, i.e. Is a separation line also a limiting streamline or is it an envelope of the limiting streamline? This question has long been discussed by prominant names such as Lighthill[30], Legendre[31], and Stewartson[11,32,34].

3.2.1 Preliminary Remarks

A clear distinction between a streamline and an envelope is conceptually important because each term has different physical meaning and implications. The task of doing this had however not been found easy. Opinions on this question vary not only among researchers, but also for a same researcher as time passes. In the latter category, two eminent names stand out: Eichanbrenner[22] was the one who first used the term "envelope," but later he[33] shifted to support the streamline version. Stewartson[32,34] used to argue for the envelope version, but recently[11] he also indicated in favor of the streamline version.

So one may wonder why this question is so complicated? There are several reasons. Ideally, to resolve this debate, one would have to investigate analytically the equation of the limiting streamlines.

$$\frac{h_2 dX_2}{h_1 dx_1} = \frac{C_{f1}(X_1, X_2)}{C_{f2}(X_1, X_2)}$$

However, since the skin frictions C_{f1} and C_{f2} as functions of (X_1, X_2) are not known in simple forms, this task is difficult even in a model study, let alone any genuine three-dimensional flows. Topological study of singularities(30,31) is conceptually interesting, but real flows could be more complex than such study suggests. Strict adherence to ideas deduced therefrom may prove to be counterproductive to new developments. This is precisely what has happened to the question of open separation. While the singularity studies fit well with closed separations, contradictions were found with open separations. On the other hand, experiments or numerical solutions provide the only concrete basis now to study separation, however the surface flow patterns obtained from these sources are usually inconclusive to settle such a delicate issue. This unsettling situation is further compounded by the fact that the terms and definitions are sometimes loosely used so that they may mean different things to different people. Worst of all, this may give the wrong impression that the whole issue is somewhat semantic.

In the following let's briefly examine the current status of this problem.

3.2.2 Envelope Version

The term "envelope" was first suggested by the superficial appearance of the limiting streamlines as revealed by the surface flow experiments. The question about whether the original definition of this term was satisfied was never seriously considered. In a differential equation book, envelope is defined as a singular solution and has usually two distinguished features (Fig. 11): (1) The envelope of a family of curves for a given equation must be tangent (at finite distance) to every curve

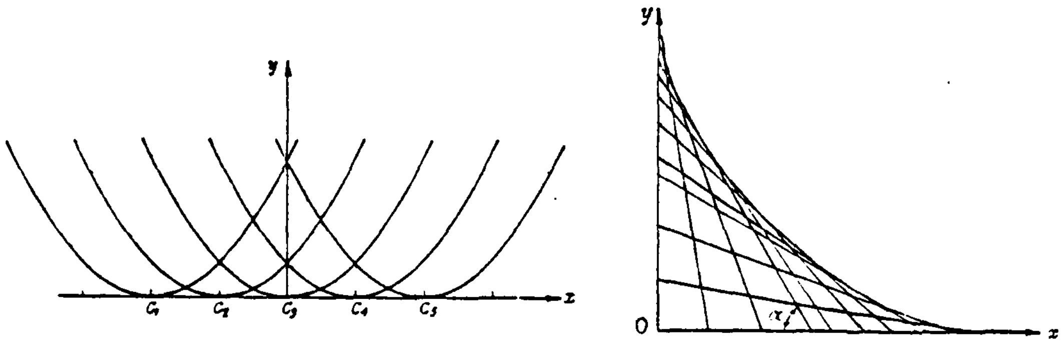


Fig. 11. Typical envelopes.

of that family, and (2) every two curves of that family intercept each other in most cases.

In the present connection, there is no easy way to demonstrate analytically whether the separation line is a singular or a regular solution of the limiting streamline equation. As to the other two features, available evidence for the first feature is not conclusive. In some cases, the limiting streamlines appear to contact the separation line at finite distance, while in others the opposite seems to be true. Admittedly the appearance of the limiting flow does not always permit an accurate judgement. The second feature is definitely not founded, the

latter fact does enhance the support of the streamline version even though these are known exceptions where envelope exists but the curves do not intercept each other[44].

In a classical work on separation, Brown[35] found at least in some particular cases, that the limiting streamlines join the separation line at finite distance and came out to support the envelope idea. Brown further found that the separation line is also a line of singularities of the boundary layer. This led to the idea of an envelope of the limiting streamlines to be connected with the idea of the Goldstein-type singularity of the boundary layer. Brown did qualify that for general three-dimensional problems, her investigations were not conclusive.

Cebeci, Khattab and Stewartson[11] recently discussed in length this streamline-vs.-envelope problem. They cited the lack of Brown's singularity in real flows as a reason for disproving the envelope version. In Sect. 4.2.5, we shall argue that this reason is open to question.

3.2.3 Streamline Version

The fact that limiting streamlines run together to form a separation line, but not intercept one and another impresses one strongly that a separation line coincides with a part of a streamline. This is especially so when the limiting streamlines do not appear to touch the separation line at a very close distance. However, there are still questions remaining to be answered before the streamline version can be convincingly accepted.

Lighthill[30] was the first one advocating the streamline idea. He identified the particular streamline which passes the singularities as

the separation line. In contrast the present author[5] has repeatedly pointed out in the past that Lighthill's definition is contradictory to our open separation idea. In an open separation, the separation line does not pass through singularities; thus one would naturally ask: (1) In what way does the separation line as a streamline differ from the rest of the limiting streamlines? and (2) is it a regular limiting streamline like the others? To the first question, the answer is likely to be that the separation line is distinguished by it being converged upon by other limiting streamlines. If this is all one can say, then the second question remains. To the latter, it is quite unlikely that a separation line will remain as a regular streamline as soon as it is at the same time converged by other streamlines, but its very nature is difficult to be spelled out. The streamline proponents have limited their considerations to what we call the closed separations. Tobak and Peake (see Sect. 4.1) did include the open separation case, but they didn't touch on the question posed here other than saying that it is approached by other streamlines. In fact, they failed to differentiate that only a part of streamline at most can be considered as the separation line as opposed to a whole streamline. The conceptual difference is while a streamline starts from the front stagnation point, the open separation line starts at the middle of the limiting flow field.

Lighthill did provide a simple model supposedly to demonstrate the streamline argument, but that model is subject to an opposite but equally valid interpretation[32,34]. Hence analytically the streamline version is not on any better ground than the envelope version. In both

versions, as far as the open separation case is concerned, the rationale relies on the appearance of the limiting flow pattern.

In short there is evidence which appears to suggest that an open separation line coincides with a part of a limiting streamline, but the very nature of that part of streamline is not quite understood yet. On the other hand, evidence does also indicate that limiting streamlines touch the separation line at finite points, the latter is a typical feature of an envelope. Experiments and numerical calculations do not unequivocally settle the delicate issue concerned here, but these provide the only basis now available for specific assessment. Analytical support of the streamline version from topological singularity studies does not apply to the case of open separation, whereas the reasoning of Cebeci et al. is still open to question. The streamline vs. envelope dispute remains to be settled clearly.

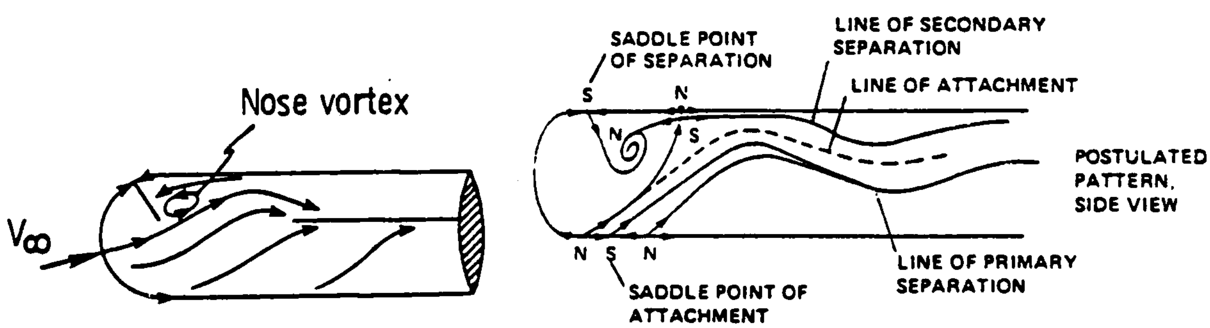
4. CRITICISM OF OPEN SEPARATION

In the past few years, the open separation idea has been challenged or criticized by Tobak and Peake[12,13,36,37] and by Cebeci, Khattab and Stewartson[11], hitherto abbreviated as CKS.

4.1 Tobak and Peake

In contradiction to our position that Lighthill's definition of separation is compatible with our closed type of separation, but is contradictory to our open type of separation, Tobak and Peake[12,36] wholeheartedly embraced Lighthill's idea. They insisted to emphasize that a line of separation in general must emerge from a saddle singularity in direct contradiction to what we have characterized the open separation. Although those authors[12,36] did not directly mention our work there[12,36] their approach is tantamount to a rejection of our open separation concept.

To stress their conceptual differences with us, they reconsidered[12] the hemi-sphere cylinder problem which we studied before[38]. Figs. 12(a)-(b) represent the respective sketches.



(a) Hsieh & Wang[38] (b) Tobak and Peake[12]

Fig. 12. Comparison of separation models.

Briefly, let us just compare the primary separation line. Fig. 12a illustrates our concept of an open separation line and is essentially the same as Fig. 1a except the nose vortex. Fig. 12b illustrates their concept. In addition to the stagnation point which is counted as a nodal attachment point, N, they added another nodal point, N, so that a saddle point, S, can be fitted into the middle. Then the particular limiting streamline which passes through this saddle point is their version of separation line in accordance with Lighthill's concept. The nose vortices and the secondary separation line in Figs. 12a, b are not of our concern here, so we shall not comment.

During the Williamsburg (Virginia) AIAA meeting in 1979 where the above paper(12) was presented, the present author commented that their approach misused the topological rules, but they disagreed. Instead, they pursued the same path in their 1980 AGARD Memorandum(36) which reappeared as NASA technical memorandum(36) later.

Later they(37) began to have second thoughts. This led them to acknowledge their error and relax their adherence to Lighthill's definitions. In Pg. 7 of Ref. (37), they stated:

> ". . . the seeming nonuniqueness of the condition identifying the particular line (as a separation line) has encouraged the appearance of alternative conditions of flow separation that, in contrast to Lighthill's, do not insist on presence of a saddle point as the origin of the line. Wang (1976), in particular, has argued that there are two types of flow separation: 'open,' in which the skin-friction line on which other lines converge does not emanate from a saddle point, and 'closed,' in which, as in Lighthill's definition, it does (see

also Wang 1974, Han and Patel 1979). In what follows, we shall address the question of an appropriate description of flow separation by an appeal to the theory of structural stability and bifurcation. Like Wang, we shall find it necessary to distinguish between types of separation, but we shall adopt a terminology that is suggested by the theoretical framework. We shall say that a skin-friction line emerging from a saddle point is a global line of separation and leads to a global flow separation. In the contrary case, where the skin-friction line on which other lines converge does not originate from a saddle point, we shall identify the line as being a local line of separation leading to local flow separation . . . ."

Thus, they abandoned their strict clinging to Lighthill's version of separation, and acknowledged the necessity of our alternative approach except they phrased their discussion in the terminology of the stability and bifurication theory. They went on in this report[37] to interpret the sequential change of a blunt-body flow versus the incidence within their perception of "super-critical bifurcation." The separation line they conceived (Fig. 13a) now turned out to be essentially the same as our open separation line (Fig. 12a), but vastly different from their prior one

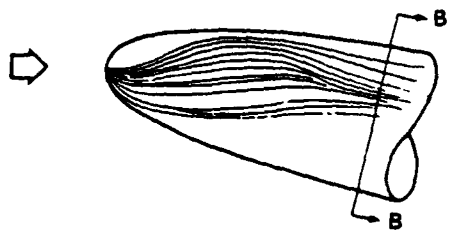


Fig. 13a. Revised separation model[37].

(Fig. 12b). The main difference is that there is only one single nodal (stagnation) point in Fig. 13a in contrast to two nodal points and one saddle point in Fig. 12b. By "essentially same" we meant that there is some conceptual difference, i.e. they still entertained Lighthill's view of a separation line to be a streamline, even though the separation line is no longer required to start from a saddle singularity. On Pg. 24[37], they remarked:

". . . We believe that this description is a true representation of the type of flow that Wang (1974, 1976) has characterized as an 'open separation.' . . . This is a case of a local flow separation."

Thus their revised separation concept amounts to a re-interpretation of our model with different terminology, justified in terms of the bifurcation theory.

On page 4 of their 1981 paper[13], they found the bifurcation theory wanting:

". . . We have been unable to discern on the basis of experimental evidence done whether any of these changes in topological structure were preceded or accompanied by asymptotic instabilities of the external flows, leading to bifurcation flows. We note that the concept of bifurcation (one flow replacing another flow that has become unstable) is principally a theoretical one; it is exceedingly difficult to confirm. . . . It may be necessary to await the further development of theory . . . before we can deduce the specific role played by bifurcation in determining the observed sequence of topological structures. . . ."

Therefore, they abandoned the bifurcation theory on the basis of which they had justified the change in our physical terminology, and invoked the idea of topological structure and structural stability as a guide in constructing possible flows.

As an illustration of their revised idea, they returned to the hemi-sphere cylinder problem(12) (Fig. 12b). Fig. 13b shows their revised concept of separation, the primary separation shown is clearly to be

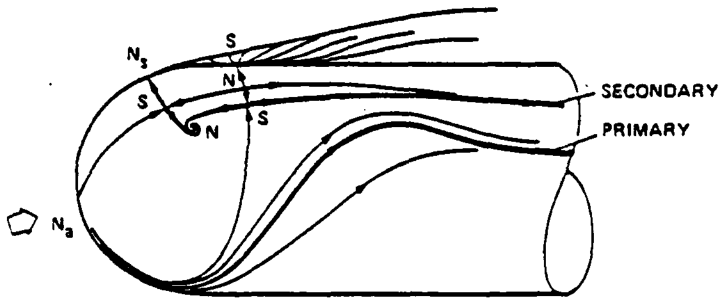


Fig. 13b. Revised hemisphere-cylinder separation(13):

similar to our open separation of Fig. 12a, but greatly different from their previous conception of Fig. 12b.

Toward the end(13), they made the following comparison between our version and their revised version of separation:

"

Open/Closed Separation versus Local/Global Separation

It remains to address briefly a topic of major concern, namely, whether current definitions of 3D flow separation are sufficiently inclusive to cover all possible cases. In our own work,[13] we have acknowledged the necessity of further refining our prior understanding of 3D flow separation by introducing a distinction between global and local flow separations. To recapitulate, we say that a skin-friction line emerging from a saddle point is a global line of separation and leads to global flow separation. In the contrary case, when the skin-friction line on which other lines converge does not originate from a saddle point we say that the line is a local line of separation leading to local flow separation. Our distinction between local and global separation bears a certain similarity to the distinction between "open" and "closed" separation introduced earlier by Wang.[24-26] It may be of interest to try to clarify the extent to which Wang's view of this matter and our own can be reconciled.

Wang's definitions of open and closed separation as presented in a recent publication[26] are accompanied by a pair of sketches showing typical examples of the two categories. The sketches are reproduced as our Figs. 8a and 8b. Wang's description of them, couched in terms of limiting streamlines, is as follows:

> The essential idea can be best explained by considering a body of revolution (Fig. 8(a), (b)) for which there is a plane of symmetry. Extension to general situations is straightforward. Fig. 8(a) illustrates an open separation; Fig. 8(b), a closed separation. Point A is the front attachment (or stagnation) point. By an open separation we mean that the separation line is not closed in the front leeside surface and does not originate or terminate at singular points in the sense that both skin-friction components vanish. The limiting streamlines on both sides of the separation line originate from the same front attachment point, i.e., the separated region is accessible to upstream flow. In contrast, for a closed separation, the separation line is closed around the body, passing through the singular points of the limiting streamlines so that the limiting streamlines on two sides of the separation line originate from two different attachment points.

By way of comparison, our own versions of the same two categories are illustrated in Figs. 8c and 8d. It is clear that Wang's description of open separation (Fig. 8a) is similar to our description of local separation (Fig. 8c); his description of closed separation (Fig. 8b) is similar to our description of a particular case of global separation (Fig. 8d). Let us first compare the description of open versus local separation. Aside from the fact that we couch our definitions in terms of skin-friction lines instead of limiting streamlines, the principal difference between our descriptions is that Wang allows the line of separation to begin, so to speak, in mid-air, whereas, with our assumption of continuous vector fields, we must insist that the line of separation be one of the infinite set of lines emerging from the nodal point of attachment and the particular one on which others of the same set converge. In comparing Wang's definition of closed separation with ours of global separation, we would criticize his use of the word "closed" as being insufficiently indicative since every skin-friction line is closed in the sense of beginning and ending at singular points. Nevertheless, Wang's sketch of a closed separation (Fig. 8b) is reconcilable with our Fig. 8d of a particular type of global separation, if the point out of which the "closed separation line" stems in Fig. 8b is identified as a saddle point. Additionally, we have included a sketch of another type of global separation (Fig. 8e) to demonstrate that the saddle point, the origin of the global line of separation, need not be associated only with nodal points of attachment, as Wang's description implies. "

Fig. 8 referred to in the quotation is reproduced here in Fig. 14. It is seen that their revised versions (Fig. 14c, d) is basically the same as ours (Fig. 14a, b) except different names. Our "open separation" corresponds to their "local separation," our "closed separation" to their "global separation." They preferred "skin-friction" line to "limiting

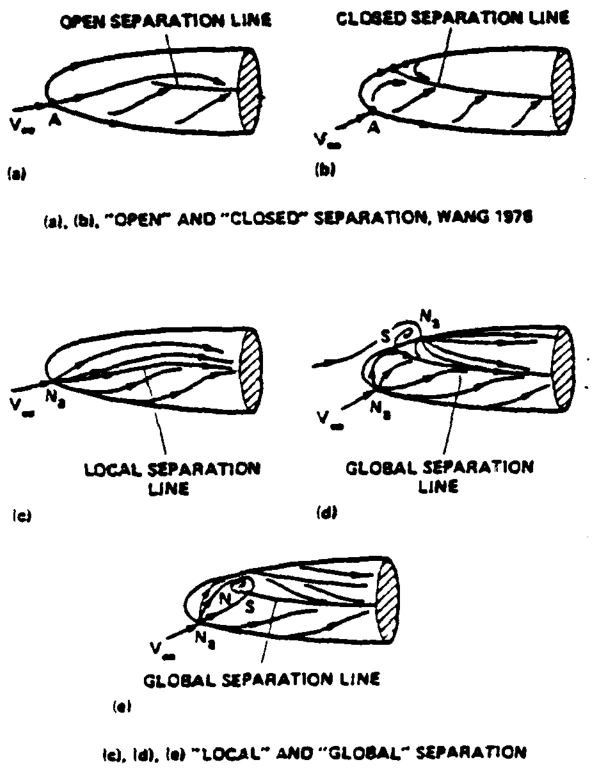


Fig. 14. Comparison of separation concepts.

streamlines." They justified their version in contrast to ours on the following grounds; they

(1) faulted our open separation line to begin in mid-air;

(2) criticized our use of the word "closed,"

(3) claimed to have found an additional type of separation in Fig. 14e and

(4) claimed their terminology, "local" and "global" having the support of theoretical framework.

We shall counter these points in the following.

(1) This question has been discussed in detail in Sect. 3.1. We disagree with their objection. On the contrary, we insist that there is nothing wrong for a separation line "to begin in mid-air," it is, in fact, an unique feature of this kind of separation. The analytical continuity argument holds for the limiting streamlines, but not necessarily for the open separation line. The latter can be considered at most to coincide with a part of (i.e. BC, Fig. 15) a particular streamline (ABC), the part

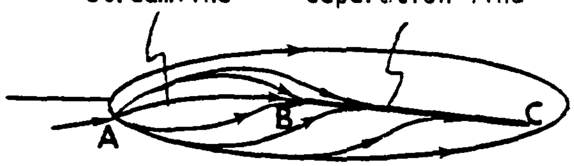


Fig. 15. Separation line vs. streamline.

AB has no meaning whatsoever with separation. Besides it is in serious doubt (see Sect. 3.2) that even the part BC to which other streamlines converge can be considered as a regular streamline. It is misleading in our view for these authors to infer the separation line to be the whole streamline ABC. Apparently although they now conceded that they have to relax Lighthall's requirement of a separation line to pass through singularities, they still attempt to cling to Lighthill's idea of a separation line being a streamline. In this way, they contended that a separation line would not have to start at mid-air which is something they objected. We consider their insistence to have such a connection is confusing. The fact is that while a streamline (such as ABC in Fig. 15) starts at the front stagnation point, an open separation line (BC) does not.

(2) The word "closed" we used is meant to signify that the separated region behind the separation line is closed to the upstream limiting streamlines. It was not meant to refer whether a limiting streamline itself is closed in the sense of beginning and ending at singular points.

(3) We are in the first place highly skeptical about the pattern of Fig. 14e to be real. Secondly, even if it were real, it still may be considered as an open type of separation because there is no indication of separation on the leeside symmetry-plane. Hence it does not represent a different type of separation as claimed. Tobak and Peake did not cite evidence of supporting this pattern which depicts a clockwise-rotating nose vortex. The latter vortex first appeared in an experiment by Werle[3] who originally interpreted the vortex to be clockwise rotating, but later concurred with the present author and Hsieh[38] that the rotation should be counter-clockwise. This idea is supported by Fig. 16 kindly provided by Dr. Werle in a private communication. Recent

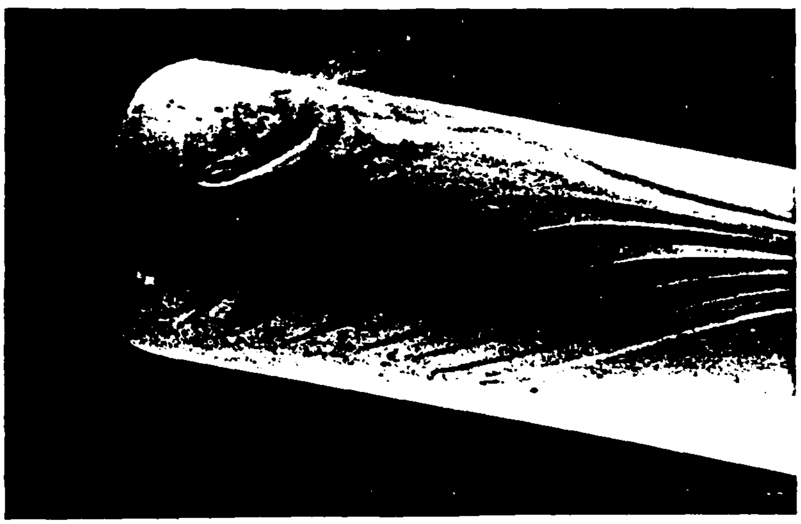


Fig. 16. Werle's experiment.

experiments unpublished yet by Dr. Bippes of DFVLR do indicate a clockwise-rotating vortex, but the separation pattern there is entirely different from that of Fig. 14e. For the above reasons, we do not think that their claim of finding an additional kind of separation is justified.

(4) Tobak and Peake borrowed the terminology of local vs. global separation from the theory of structural stability and bifurication. The latter subjects are highly theoretical, there is no evidence what these have to do with various separation patterns. Until those authors convincingly demonstrate this connection, their revised separation concept amounts to little more than a re-interpretation of our model with a superficial change of terminology. By global separation, it must mean that the separation has large overall effects. By the same logic, a "local" separation must mean that the separation effects are more confined in extent. Actually[5,27] an open separation generates larger wake behind, while the wake behind a closed separation is more confined. On this basis, there appears to have a contradiction of terms by renaming "open" as "local" and "closed" as "global."

In any case, it is gratifying that Tobak and Peake "refined their prior understanding of 3D flow separation" and thereby came to a position essentially same as ours. Unfortunately they still feel necessary to introduce a different terminology, and continue to infer a separation line to a whole streamline. Their experience in this case must have cautioned us that topological study alone does not guarantee to determine separation correctly.

4.2 Criticism of CKS

Another objection to our open separation idea has been raised by CKS. They acknowledged that open separation occurs in many practical flows, but they objected to it on the grounds that the flow on the separated leeside of the body is uncomputable according to the boundary layer theory. We shall first summarize below their work and then respond to several questions involved.

4.2.1 CKS' Calculations

Those authors reconsidered the same problem we studied before, i.e. the laminary boundary layer over an ellipsoid of revolution with a minor and major-axis ratio being equal to 1/4. We calculated the cases[25-27] of the incidence α being 6°, 30° and 45° to represent three different stages of flow separations; i.e. closed separation at low incidence (6°), open separation at moderate to high incidence (30°) and closed separation again at high incidence (45°). They calculated the cases of α = 6°, 15° and 30°.

Their procedures of calculation are also the same as ours, for example: (1) Computation marches from the windside upward and from the leeside downward, and (2) using the standard box scheme for the windside up to where the circumferential flow reverses and using thereafter the zig-zag or characteristics box scheme for reversed region circumferential-flow. Their standard box scheme corresponds to our scheme 1[25-27] and their zig-zag or characteristics box scheme corresponds to our scheme 4. They introduced a transformation which avoids usual difficulties near the vortex of the body.

Their results of 6° and 30° incidence are essentially the same as ours. Their 15° results are of the same character as those of 30°, hence carry no particular physical significance. Since open separation is the main theme of this report, we choose to compare briefly the results at high incidence. Fig. 17a shows their calculated separation for α = 30°, the letters A, B, C are added by this author for convenience of discussion. AB is marked the separation line, AC the accessible line. They

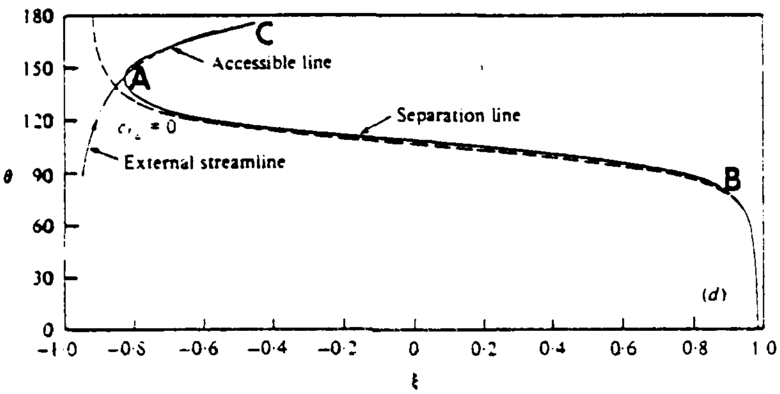


Fig. 17a. Calculated separation by CKS, α = 30°.

confirmed our finding that the zero $c_{f\theta}$ line is very close to the separation line. Our results[26] for the α = 30° problem are shown in Fig. 17b.

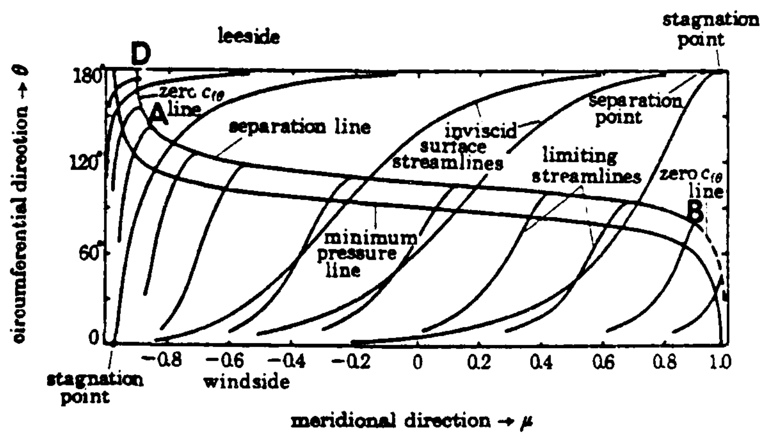


Fig. 17b. Calculated separation by Wang, α = 30°.

We remarked in Ref. 26 that although the separation line "is located slightly above the zero-$c_{f\theta}$ line, we will not make such a distinction because they are so close to each other." Hence in Fig. 17b, the separation line AB is shown to coincide with the zero-$c_{f\theta}$ line, except the part AD (dotted line) which is marked as the zero-$c_{f\theta}$ line and has no meaning of separation. In Fig. 17c, the zero-$c_{f\theta}$ lines from two calculations are seen to be identical.

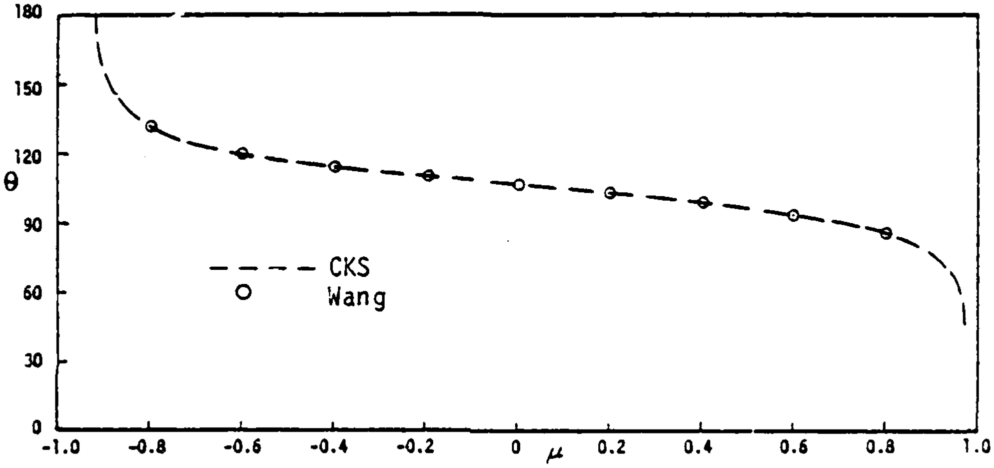


Fig. 17c. Comparison of the zero-$c_{f\theta}$ line, $\alpha = 30°$.

They claimed that their leeside results of 30° extend much beyond what we reported before. Actually not only such extension covered a negligible area over the body surface, but also similar extension was reported on a separate occasion(39) by this author. The skin-frictions reproduced in Fig. 18 (originally Fig. 25 in Ref. 39) show that calculation broke down at $\mu = -0.83$ and $\theta = 140°$ which are the same values given by CKS. Having demonstrated that the calculated results are essentially the same, it is evident that disagreements arise mainly from different interpretations and other related considerations.

(a)

(b)

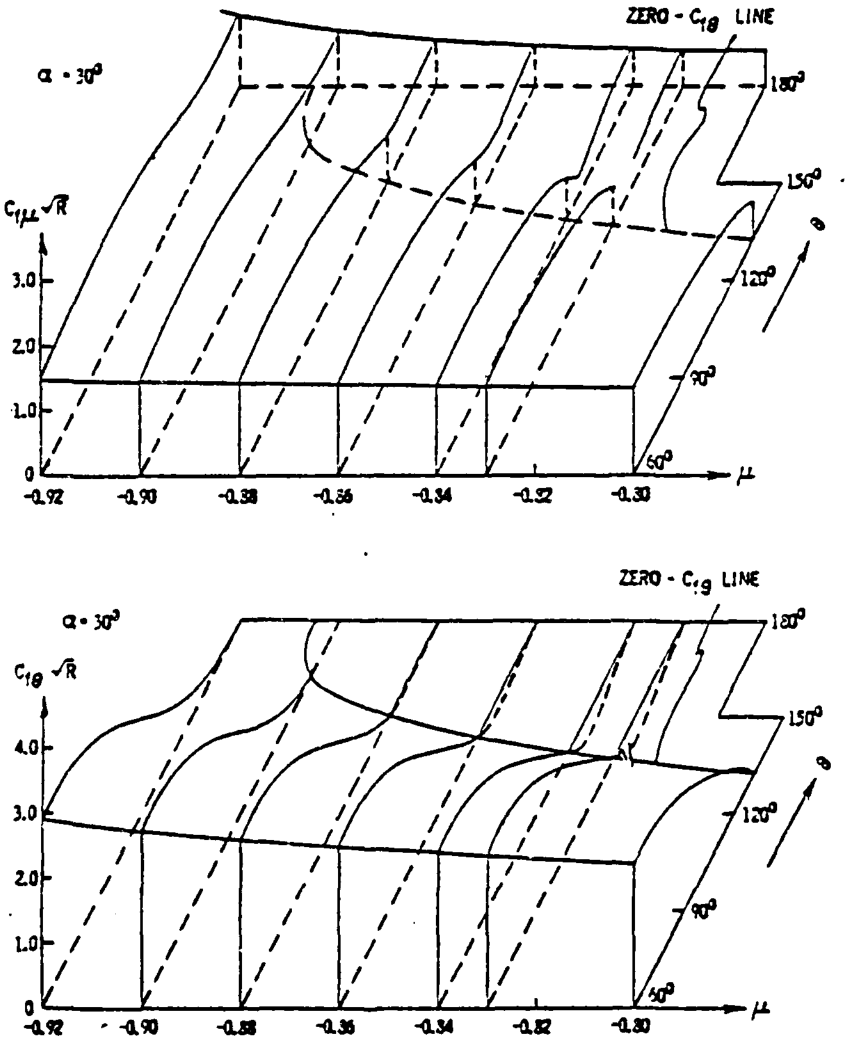


Fig. 18. Extended calculation by Wang[39],
(a) Meridional skin friction, $c_{f\mu}$,
(b) Circumferential skin friction, $c_{f\theta}$.

4.2.2 Box Schemes

The numerical schemes used by those authors were claimed to be superior to others' including those used by this author. The truth of such overclaim needs to be clarified. It is a fact that the crank-Nicolson scheme which we used leads to a much simpler tri-diagonal matrix than that resulting from the box scheme they used. More important is how the reversed flow will be treated. For the latter purpose, those authors employed two such schemes: the zig-zag box and the characteristic box. The zig-zag box scheme used in this work was used by Cebeci before in an unsteady calculation. As was pointed out by this author[16] in the latter connection, this method is incorrect because of its failure to satisfy completely the zone of dependence. For a reversed profile across ABC (Fig. 19), they employed their standard box scheme for the part AB and

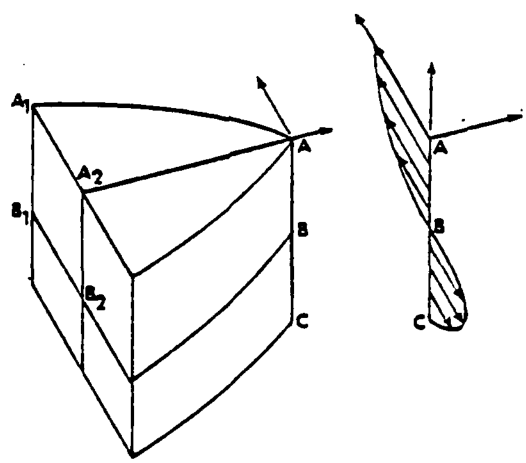


Fig. 19. Dependence zone.

their zig-zag scheme for BC. If the zone of dependence is divided into four quarters, their procedure amounts to the quarter $ABA_1B_1A_2B_2$ being neglected. This probably explains why this approach does not entirely

mimic the wave-like character of the momentum equation as these authors noted themselves. The correct procedure requires that whenever reversed flow occurs, such zig-zag method must be used across the entire layer AC instead of just across BC. The latter would have been correct if the problem were purely hyperbolic. The diffusion process in the normal direction changes the picture for the present boundary layer problems.

Another scheme used by those authors--the characteristic box scheme--did avoid the just mentioned defect, because it was applied across the entire layer AC (Fig. 19) rather than just across the lower segment BC. They reported that the results from the characteristic box scheme are more accurate than those from the zig-zag box and attributed this to the advantage of the characteristics lines being followed in their calculations. In our opinion, this is unlikely the correct interpretation. We contend that the real reason is that in their characteristic box method, the dependence rule was satisfied whereas in their zig-zag box method, the same rule was not. Furthermore, the characteristics box scheme is more complicated than necessary. As long as the dependence zone is enclosed by the computation mesh, accurate solutions can be obtained by following more convenient coordinates than the characteristics. This has been demonstrated long before in the calculation of the classical supersonic flow where the idea of the dependence zone was first advanced. The experience of testing the characteristics box scheme at Royal Aircraft Establishment as reported by Smith[40] supports our above assessment.

4.2.3 New Definition of Accessibility and Separation

Accessibility was used to be defined by Stewartson[32] to mean that "a point P on the boundary is said to be accessible if particles of fluid arbitrarily near P originally came from the neighborhood of attachment." In other words, accessibility means whether a point can be reached by one of the limiting streamlines from the front stagnation point. It was in this sense, the present author commented[5] that inaccessibility cannot be used as a criterion of separation, because it contradicts to the very meaning of an open separation.

Presumably in response to the above comment, CKS returned to this theme in their paper[11] considered here. They introduced a new definition of accessibility: "a point P of the boundary layer is said accessible from the forward stagnation point of 0 if the velocity of P can be computed in terms of the initial conditions at 0 and the boundary conditions on the body and in the external stream."

There are obvious differences between these two definitions. The old one speaks of a point on the body surface, the new one is concerned with a point of the boundary layer including those above the wall. The old one follows the physical meaning of term, i.e. whether a point can be reached by one of the limiting streamlines, the new one stresses whether the velocity of a point can be calculated according to the classical boundary layer procedures. The velocity of a point on the body which is accessible by a limiting streamline may not also be computed according to the boundary layer procedure. Hence a point which is accessible according to the old definition would become inaccessible according to the new

definition. The points above an open separation line (see Fig. 17b) are precisely such examples.

On the question of separation, they defined "separation is a line l_s drawn on the body which forms the boundary of all limiting streamlines emanating from 0 (meaning the front stagnation point). If l_s is closed, the normals to it bound the region of accessibility but to identify l_s and l_a requires additional arguments, although Wang (1976) noted a general agreement that it is always the case. Later on, however, we shall demonstrate that important exception can occur."

Compared to the more conventional description of separation (Sect. 1.3) as the running-together of the limiting streamlines, CKS' new definition of separation is just a slight variance which not only lacks fresh ingredient, but also is less specific and less general. Both versions rely on the behavior of the limiting streamlines, and both leave the question of being a streamline or an envelope unanswered. The conventional one specifies the feature that the limiting streamlines run together or converge to the separation line, whereas the new one only says it forms the boundary of the streamlines. The new one is less general because the requirement of the separation line being the boundary of <u>all</u> streamlines virtually rules out the case of open separation as we shall explain below. Their reference to this author's position in the above quotation is very misleading. First of all, we never talked about a line of accessibility. Secondly, their new definition of accessibility is different from that we used[5]. Thirdly, also more importantly, we[5] did not agree that accessibility is the proper criterion for separation, because it holds for closed separations but not for open separations.

The emphasis that l_s "forms the boundary of all limiting streamlines emanating from 0" again fits well with the closed separation idea, but not that of an open separation. In the latter case, even the primary separation lines on two sides of an inclined body of revolution are not connected, hence not a single curve on the body, let alone the additional secondary separation lines. Therefore, a part of the limiting streamlines from 0 approaches the top side of the primary separation line, another part approaches the bottom side of that same line. Furthermore, part (of the limiting streamlines) approaches the primary separation line, part approaches the secondary separation line. Thus CKS' new notion of separation virtually exclude the existence of open separation. For this reason, it is not as general as that we discussed in Sect. 1.3 which applies to both closed and open separations.

4.2.4 Separation Dispute

(a) Low Incidence. Their calculated separation line for the 6° incidence is of same tongue-like shape as we reported[25]. Hence there is no separation quarrel for the low incidence.

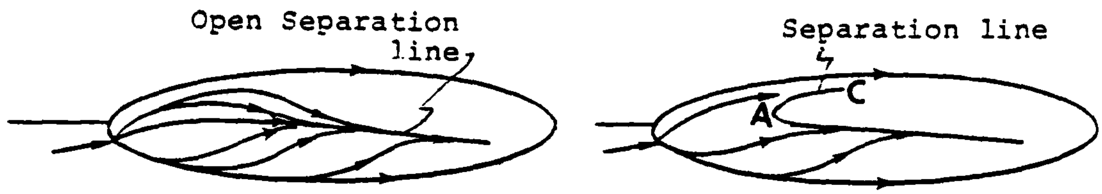
However, in connection with this 6° case, they remarked on p. 61 "In contrast to Wang's results, we find that the circumferential skin friction does not vanish on l_s and nor does the tangential component on l_s." We consider this inference is a misrepresentation of our position. We never said that the circumferential skin friction $c_{f\theta}$ vanishes on the separation line at low incidence (say 6°). On the contrary, we emphasized[25] again and again that a significant area of circumferential flow reversal was calculated for the first time ahead of the separation line

(see Fig. 12a of Ref. 25). Se we were in fact the first to demonstrate specifically the distinction between the zero-$c_{f\theta}$ line and the separation line. The fact that the tangential skin friction does not vanish on the separation line is so elementary for this subject matter, we wonder where did these authors get the idea that we ever said to the contrary. What we did say[26] was that for high incidence (say $\alpha = 30°$), the separation line is very close to (though slightly above) the zero-$c_{f\theta}$ line and hence for all purposes, the position of the zero-$c_{f\theta}$ line can be taken same as the separation line. On this point, CKS in fact reached the same conclusion. Meanwhile we did carefully point out (p. 49, Ref. 26) that "A zero-$c_{f\theta}$ line does not always imply separation. . . . Also at low incidence . . . , the whole zero-$c_{f\theta}$ line has no implication of separation at all."

(b) High Incidence. On p. 83, they stated ". . . the OK region of l_s is difficult to be definite about . . . . In this case also our inclination is favor Fig. 10a that l_s is closed in this neighorhood but the evidence is very weak. Wang (1976) favors open separation at $\alpha = 30°$ over a substantial proportion if not all of the leeward part of l_s. In our view the notion of inaccessibility prompts a definite decision on this question." The Fig. 10a they referred to depicts a pattern essentially same as that of Fig. 20b. This quotation contains the essence of the present dispute. It was made in connection with discussions about $\alpha = 15°$, but it was understood that the same applies for other high incidence such as $\alpha = 30°$.

Two issues stand out: (1) They contended that the tongue-shaped pattern at low incidence prevails at high incidence even though they

admitted that "the evidence is very weak." Thus their conceived separation pattern (Fig. 20b) will be very different from our open separation



(a) Wang

(b) CKS

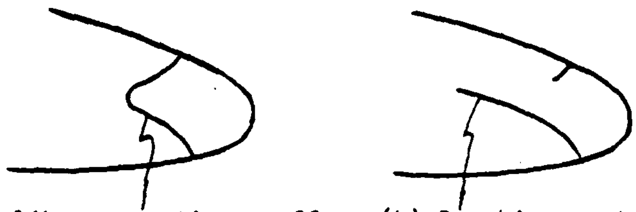
Fig. 20. Comparison of separation model.

(Fig. 20a). (2) Because the leeside flow can not be calculated according to the boundary layer theory, they objected to our open separation idea.

With respect to the first issue, let us point out here that even the pattern of Fig. 20b were correct, the open separation view would still hold because the pattern is still open near the symmetry-plane. So the idea of openness is not in question. The only question is concerned with whether the separation line consists of the part AC? or in other words, whether the separation line has turned around?

The last posed question is further connected with how the tongue-shaped separation determined at low evidence would change with increasing incidence. This question is still not well understood at this time. This author first[5] conjectured that such shape would break up (Fig. 21b). This is mainly motivated by our anticipation that the lower branch will gradually extend forward to become an open separation line (see Fig. 20a). But there was no basis to say what might happen to the upper branch because little was (is) known about the separated flow over the

aft body. Our speculation now is that the upper branch might also



(a) Tongue-like separation at 6°   (b) Breaking-up to form an open separation

Fig. 21. Possible development of tongue-like separation

extend forward to start forming the secondary separation line. The secondary separation line in fact is of the open type (see Figs. 3, 8 thru 10). It must be differentiated that although we do not know for sure how the tongue-shaped separation will evolve, yet we have asserted all along that the separation at high incidence is of an open type (Fig. 20a or 1a). Those two are separate questions and should not be tied together.

On the other hand, CKS favored the tongue-like pattern to persist at high incidence (Fig. 20b). There are two objections: (1) their evidence about this tendency according to their calculation is very weak as they noted themselves, and (2) more importantly, this tendency has not found any support from experiments or other calculations.

The second issue is concerned with whether we can reach the conclusion of an open type of separation while the leeside flow could not be calculated according to the boundary layer theory.

To answer this question, let us recapitulate the basis on which the open separation idea was initiated: (1) We know first from the symmetry-plane boundary layer investigations[2,24] that there is no separation along the symmetry-plane ahead of the point S (Fig. 22) which

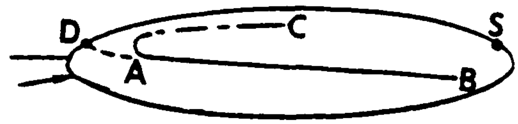


Fig. 22. Illustrating the open separation.

is located close to the rear end. (2) It is inconceivable intuitively that for a typical elongated body of revolution at high incidence, the separation would be confined to the very rear end of the body. The high adverse pressure gradient along the circumferential direction must cause separation on two sides (say along the line AB in Fig. 22) of the symmetry-plane. This leads inevitably to the very idea of open separation in the sense that the separation line is not connected to the symmetry-plane plane at the forebody as indicated by the dotted line AD. (3) The location of the open separation line AB was determined by three-dimensional boundary layer calculations. (4) This whole idea is consistent with experimental observations, some old interpretations of the experiments contrary to this idea were argued to be wrong. (5) The question about whether the separation line may turn around and have the portion AC was less obvious, we ruled out this possibility partly on the basis of our result (Fig. 7b) which gives no such indication and partly on the observation of surface-flow experiments.

Back now to the second issue, does the fact that the leeside solution cannot be obtained by the boundary layer calculation really prevent us from establishing the open separation idea? We maintain our

determination of the open separation idea has been carefully examined from different angles, and particularly the conclusion has been confirmed by all later available evidence, experimental or computational. CKS insisted to demand a leeside boundary layer solution in order to "legitimatize" our claim, this is impossible because such solution does not exixt, and the boundary layer approximation simply is not valid there.

It should be reminded that we did complete the windside solution up to the separation line. This is precisely one has always done in two dimensional investigations, i.e. calculate up to the separation point while leave the separated area undetermined. Has anyone ever demanded that the latter must be solved before speaking about two-dimensional separation?

On p. 61, CKS stated "In fact the concept of separation is strictly meaningless here and the notion of open separation introduced by Wang (1975) is irrelevant, although no doubt important in many practical flows." We consider this statement to be rather misleading, confusing and out of place. We don't see the logic why the concept of separation becomes meaningless just because the leeside separated flow can not be calculated by the boundary layer theory? Why our open separation idea is irrelevant whereas the same idea has been overwhelmingly supported by all others (see Sect. 2)? They (CKS) discounted the open separation idea as irrelevant, but admitted also its importance "in many practical flows." So they seemed to imply that separation according to the boundary layer theory is different from that in real flows? In other words, they implied that there are different kinds of separations, one according to the boundary layer theory and another in real flows or according to higher

approximate theories. If this were the case, then what is the point to study separation according to the boundary layer theory since it is so basically different from that in the corresponding real flow.

On p. 85, they stated "Wang infers that the leeside of l_s coincides with the windward side of l_s . . . and that is an example of open separation." We consider this quotation provides another example of their misinterpretation of our position. We deny that the way they attributed to us was how we reached the idea of open separation. We never thought of at high incidence that there is a continuous separation line which turns around and has the leeward and windward parts similar to that at low incidence. Consequently we never conceived or inferred that the open separation line results from those two parts coinciding together.

Also on p. 85, they contended that our claim of open separation is "not legitimate for it is quite impossible to integrate the equations on the leeward side far enough towards the windward side to reach l_s." If their notion of legitimacy means that the flow on the leeside of the open separation line must be calculated as opposed to reference to experiments, we would like to point out that such examples not only exist but also fully support our open separation idea. These are the work of Rizk et al.[14] and of Hsieh[29] which employ the thin-layer approximation, and allow the pressure to be calculated along with the velocities and and hence include the viscous-inviscid interactions.

4.2.5 Streamline vs. Envelope

CKS devoted a good deal of discussion to the streamline vs. envelope question. In essence, they asserted that their results

definitely suggested their windside separation line being a Brown[35]-type of envelope, but for their leeside counterpart at low incidence no definite conclusion could be given. At the meantime, they indicated their inclination of favoring the streamline version at several points of their paper[11]. Our position on the streamline vs. envelope question has been discussed in Sect. 3.2, here we only intend to comment on CKS' reasons for their support of the streamline version.

On p. 58, they stated "For, if it is an envelope, the skin friction component in a direction perpendicular to l_s must have an algebraic singularity . . . and so the solution terminates at l_s. On the other hand if l_s is a skin-friction line then it is possible for the solution to be smooth at l_s and so be continued beyond . . . although, to be sure, additional information must then be supplied to specify it uniquely." We consider such a way to characterize the difference between the streamline version and the envelope version is very misleading. Whether solution can be continued across the separation line does not rest with how one names the separation line, rather with the method by which one would actually solve the problem. The catch here is the qualification "additional information must be supplied." Once additional information must be supplied, the problem will no longer be the original boundary layer problem, it would very likely change from a parabolic type to an elliptical type. In that case, it is expected that solution can be continued smoothly across l_s. The algebraic singularity they referred to is strictly associated with the boundary layer formulation, consequently if additional information is added and the problem changes its character, then this singularity will also disappear.

Similar sentiment was repeated later in the paper[11]. On p. 78 they stated "Experimentally inspired statements about l_s are therefore not germane to the question for in a practical flow the solution must be regular and Lighthill's notion of its being a limiting streamline is correct." Again on p. 83, "An immediate consequence is that the singularity at l_s on the windward side is prevented and from that it follows that l_s can not be an envelope of limiting streamlines. Thus for a real flow Lighthill's (1963) concept of l_s as a limiting streamline is relevant."

Thus they argued that the absence of singularity in real flows contradicts the envelope version and lends support to the streamline version. We consider these assertions are open to question regardless which version would eventually prevail.

The central point is whether the envelope of the limiting streamlines is necessarily tied with Brown's singularity so that these two would imply each other in general. In our view, there is no proof that this is true. Brown's investigation was based on the boundary layer equations, but CKS applied her finding to real flows and to higher approximations of theory in general. It is quite likely that should Brown have employed Navier-Stokes equations in her analysis, the singularity might not have appeared while the separation line still has the appearance of an envelope (on the basis that the limiting streamlines touch it at finite distance). Hence, the absence of Brown singularity in real flows does not invalidate the envelope version.

Likewise, the relevance of the streamline version does not necessarily follow from the absence of Brown's singularity. In light of the open separation idea, we insist that Lighthill's definition of separation

must not be accepted as such. As mentioned in Sect. 3.2, even if we accept that a separation line can be looked at as coinciding with a part of a particular streamline, it is still doubtful that this part of that particular streamline is of the same nature as the rest of limiting streamlines.

In short, although the time to call in the streamline vs. envelope dispute is not ripe yet, we don't think CKS' reasons for the streamline version are convincing. The connection between Brown's singularity and the envelope is strictly for the boundary layer theory. Lack of singularity in real flows does not constitute as a basic to refute the envelope version.

We shall now sum up our response in Sect. 4.2. CKS' calculations merely confirmed our earlier results. Their claims about the extension of calculations and the superiority of numerical schemes are not justified. Their zig-zag box scheme was not correct because it did not completely satisfy the zone of dependence rule. Their characteristics box scheme not only is more complicated than necessary, but also offers little advantage compared to other schemes so long as the dependence rule is satisfied. Their new definition of accessibility seems to have led them to reach only an obvious conclusion: i.e., the flow on the leeside of our open separation can not be calculated within the framework of the boundary layer theory. They based on the latter fact to object our open separation idea. They termed our open separation idea as "irrelevant" and "illegitimate," whereas the same has been confirmed by all related experiments and calculations. If their notion of "relevance" and

"legitimacy" means complete calculations rather than partial appeal to experiments, such a solution is in fact not only available from a thin-layer calculation, but also fully supports the open separation idea. They contended that open separation happens in real flows but not according to the boundary layer theory, implying that there are two different kinds of separation, one for real flows and another for the boundary layer theory. We consider such an implication is misleading. Separation is a unique physical phenomenon, no matter which valid theory one may use to predict it, the end results must be consistent in basic characters if not in numerical values.

Their new definition of separation contains little new ingredients, and is, in fact, less general than the commonly used one as we described in Sect. 1.3. Their preferred separation pattern (Fig. 20b) is neither supported by their own calculated results, nor by any available experimental evidence.

Their discussion on the streamline vs. envelope dispute did not end with definitive conclusions. Their calculated results asserted their windside branch of separation line (i.e. our open separation line) to be of an envelope nature, but otherwise were inconclusive in general. The Goldstein singularity found by Brown in connection with the envelope was strictly for boundary layers, but CKS overextended it to flows in general. They argued that lack of singularity in real flows implies supporting the streamline version while repudiating the envelope version, we maintain that both these two assertions are open to questions.

5. EXTENSION TO UNSTEADY FLOWS

Due to the similarities between the equations of three-dimensional, steady boundary layers and those of two-dimensional, unsteady boundary layers, several ideas developed for the former may be analogously carried over to the latter. The rule of the zones of dependence and influence[41] was an example in this light, here we give another example related to separation.

The open and closed separations developed for three-dimensional, steady boundary layers are determined by the running-together of limiting streamlines on a body surface (Fig. 1). To carry this idea to two-dimensional, unsteady cases, one can define an analogous set of limiting lines in a x,t-plane[41] where x is measured along a two-dimensional body, and t the time. Similarly unsteady separation may also be classified into open and closed types. An open unsteady separation occurs when there is no separation initially, but separation develops at later times (Fig. 23a).

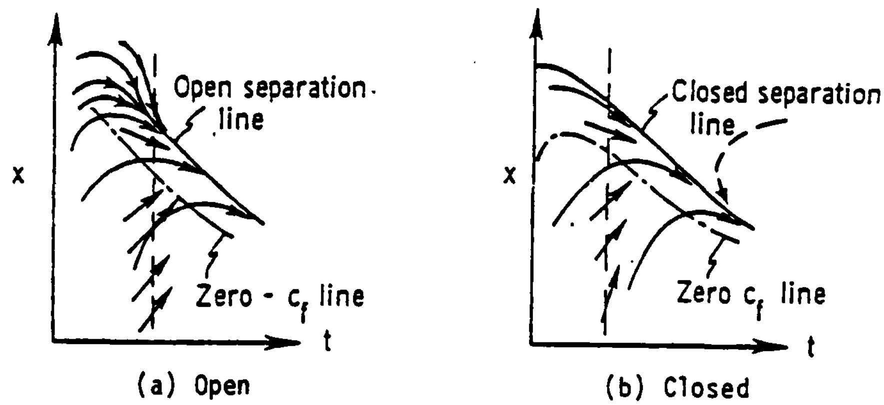


Fig. 23. Analogous unsteady separation.

On the other hand, a closed unsteady separation occurs when separation starts at the very beginning of an unsteady motion (Fig. 23b). Like the steady counterpart, unsteady open separation means that the limiting

streamlines may reach both sides of the separation line, while in a closed separation, the limiting lines will be confined to the same side of the separation line, i.e. the lower side in Fig. 23b. Unlike in the three-dimensional steady case where reversal along both surface directions is common, events can not be reversed in time. Hence the trend depicted by the dotted limiting line above the separation line in Fig. 23b is not possible.

In Ref. 15, two examples were calculated, one each for the open and closed separation respectively. Fig. 24 shows the calculated limiting streamline pattern in the θ,t-plane for the classical problem of an

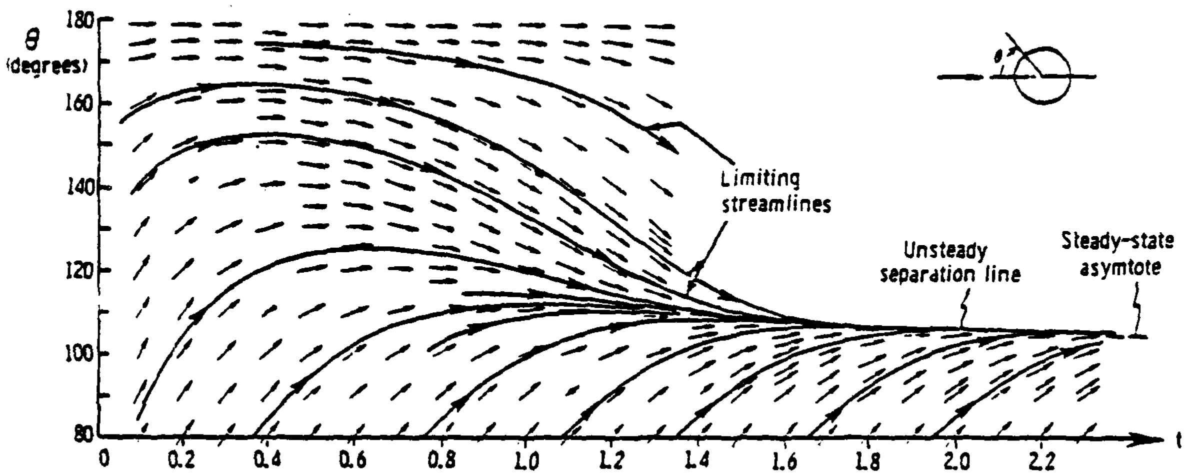


Fig. 24. Unsteady limiting flow pattern and separation line in the θ,t-plane.

impulsively-started circular cylinder. The limiting lines are seen to run together at later times, this is identified as the unsteady open separation line.

The above mentioned unsteady separation for a circular cylinder was once a controversial problem[16]. Our prediction of separation based

on the above analogy method was found in agreement with the result of Von Dommeleu and Shen[45], but was contradicted by Cebeci, Khattab and Stewartson[42,11]. The latter authors maintained that such unsteady separation is impossible unless there is separation at the very beginning. However, this dispute was ended when Cebeci[43] reported that his early conclusion was in error.

The open and closed separation idea has also been recently extended[17] along a different direction to unsteady flows in three dimensions. For the steady flow over an ellipsoid of revolution (b/a = 1/4) at high incidence ($\alpha = 45°$), the separation is known[27] to be of a closed type (Fig. 25c). The question concerned here is how such a closed

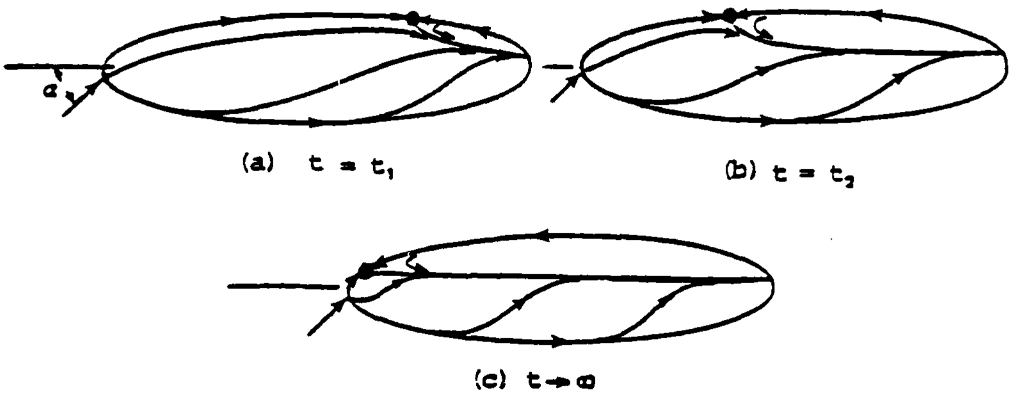


Fig. 25. Conventional sequence of unsteady separation.

separation is developed in an unsteady process if the same body is impulsively started from rest? Intuitively one would be inclined to think that such unsteady growth will go through the stages as depicted in Fig. 25a,b. In other words, the separated region continues expending forward, but a closed separation prevails at every intermediate instant.

However, our recent calculation of the symmetry-plane boundary layer over such an impulsively-started ellipsoid of revolution at 45° incidence suggested otherwise. The skin friction along the leeside symmetry-plane resulting from this calculation is shown in Fig. 26. It is seen that the zero-skin-friction point remains at the rear end when the

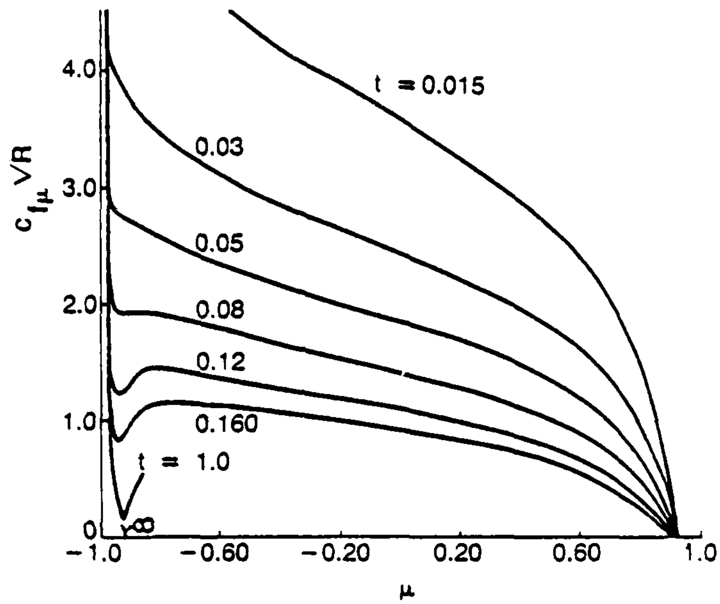


Fig. 26. Variation of the skin friction along the symmetry plane.

incidence increases, but there develops a minimum point on the skin friction curve and such minimum point dips slowly with increasing time. Similar behavior was previously found in the steady case[2,24], only the varying parameter there is not the time, but the incidence. It was precisely this unexpected feature which led us to conceive the idea of an open separation[1,5]. Following the same reasoning, it was suggested[17] that most likely the unsteady separation sequence develops as depicted in Fig. 27, i.e. an open type of separation prevails throughout the entire growth process, while the closed type of separation is formed only at the final steady condition.

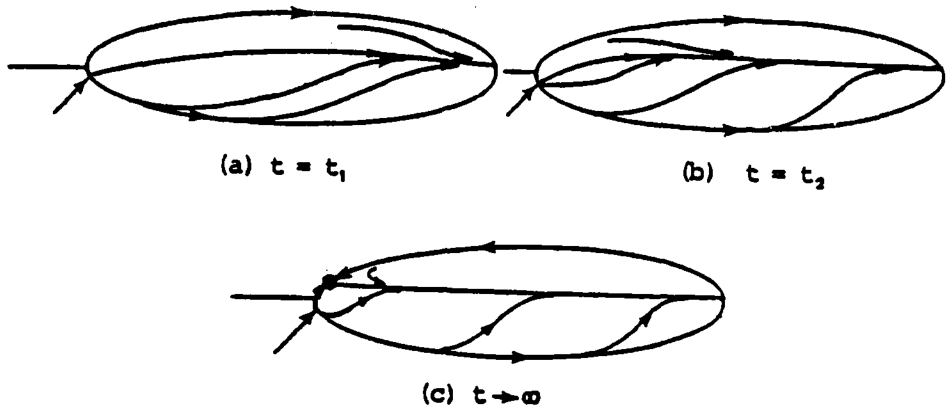


Fig. 27. Proposed sequence of unsteady separation.

It should be made clear that the above discussion holds only for the high incidence case. The unsteady separation sequence for the low incidence case is expected to be different and is being investigated.

6. CONCLUSIONS

After a decade since its inception, the open separation concept has gained wide acceptance, additional confirmations from both experiments and calculations continue being reported in the literature. Its application has been extended from steady to unsteady cases, from aerospace area to automobile design.

Separation in three-dimensional flows can be in general identified by the running-together of the limiting streamlines. This is so regardless whether the separation line is a streamline or an envelope.

New developments since 1976 overwhelmingly lend support to the open separation idea. The basic question of where an open separation line starts is convincingly demonstrated by results from experiments and calculations. It starts in the middle of the surface flow field rather than a saddle singularity as usually assumed. Because its direct contradiction to the conventional concept, this single issue has been a stumbling block for many to accept the open separation idea.

Another long-debated issue about whether a separation line is a streamline or an envelope is examined but without definite conclusion. Support for both versions can be cited from available experiments and calculations. In principle, analytical study is more suitable for resolving such a delicate issue, but this is not an easy task because of the complexity of the problem involved. Topographical study, however interesting, has been found to be inapplicable to open separations, whereas other analytical reasonings are still open to questions. In spite that there are researchers who used to support the envelope idea, but have

later changed their mind to favor the streamline version, it does not seem to this author that the time to make such a call is ripe yet.

Contradicting to our position that Lighthill's concept of separation is compatible with our closed separation but not with our open separation, Tobak and Peake at first wholeheartedly embraced Lighthill's idea and insisted it to be valid in general. However, they reversed their stand later and thereby came to a position essentially same as ours, although they rephrased in terms of a different set of terminology. Until they could demonstrate convincingly the connection between separation and flows structural stability, otherwise their version of separation amounts to little more than a superficial change of names with no new substance. Meanwhile it is still misleading for them to continue relating an open separation line to a whole streamline.

Various questions raised by CKS are replied in details. There are unjustified overclaims, misunderstandings and misrepresentations of our positions. Their calculated results revealed little new compared to ours, so the differences arise entirely from different conceptions. Their claim of superior numerical schemes is unfounded. Their new definition of accessibility does not bring out new physical insights, but makes the problem to appear more abstract and mathematical compared to the old definition. It proved in the end an obvious conclusion that the flow on the leeside of our open separation line can not be calculated within the framework of the boundary layer theory. For this very reason, they termed our open separation idea to be "irrelevant" and "illegitimate." Their demand to calculate the leeside separated flow (as just mentioned) in order to legitimize our position is impossible to fulfill because the boundary

layer theory is known to be invalid there. On the other hand, our open separation idea has been confirmed not only by all related experiments, but also by more complete solutions (i.e. more complete than boundary layer solution) using the thin-layer approximation. CKS admitted that open separation is a real phenomenon in real flows. The implication that the separation of boundary layer for a given problem is different from that of the actual flow is misleading.

The idea of open separation developed for three-dimensional steady boundary layer can be analogously carried over to unsteady flows in two or three dimensions. In the two-dimensional, unsteady case, this was demonstrated in a classical problem of a circular cylinder started impulsively from rest. For the three-dimensional unsteady case, the investigation has so far been limited to the symmetry-plane boundary layer problem, but the results obtained for a particular case (i.e. an impulsively-started ellipsoid of revolution at high incidence) have surprisingly far-reaching implications to three-dimensional unsteady separation in general. This investigation is still in progress.

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